Nuclear Resonance Spectroscopy: Opportunities with New Radiation Sources

Gopal Shenoy
Argonne National Laboratory

Symposium
50 Years After - the Mössbauer Effect Today and in the Future
Technische Universität München, Physics Department,
Garching (Germany), 9-10 October, 2008
Acknowledgements: Some of the early pioneers at Argonne and to a few hundred collaborator and friends
DOE/BES for Funding the Research
Outline

- Recall Third Generation Sources
- Introduction to Next Generation Sources
- Opportunities in Materials Physics
- Dreams and Speculations
Why Synchrotron Radiation Sources?

Nuclear resonance and brightness of synchrotron radiation

Undulator based SR

10 mCi $^{57}\text{Co}$ source

$10^{23}$ photons / sec / eV / sr

$10^{11}$ photons / sec / eV / sr

Eric Gerdau et al.
PRL 54 (1985)


Z | Isotope  | $E_o$ (keV) | $\tau_o$ (ns) | $\Gamma_o$ (neV) | $\sigma_o$ (kbarn) | $\sigma_o/\sigma_{el}$
---|----------|-------------|--------------|----------------|-------------------|------------------
26 | $^{57}$Fe | 14.4125     | 141.1        | 4.7            | 2464              | 426              

Characteristics of nuclear excitation and decay

$^{57}$Co

$t_{1/2} = 270$ days

Electron capture

Spin, I

Energy

5/2

91%

136 keV

3/2

9%

14.41256 keV

1/2

9%

0

$Z = 26$, $E_o = 14.4125$ keV, $\tau_o = 141.1$ ns, $\Gamma_o = 4.7$ neV, $\sigma_o = 2464$ kbarn, $\sigma_o/\sigma_{el} = 426$
Incoherent Nuclear Resonant Inelastic X-ray Scattering (NRIXS)

Seto et al. PRL 74, 3828, 1995
Sturhahn et al. PRL 74, 3832, 1995

1 meV ~ 0.66 ps
Overview: 3rd generation sources
What Are the Requirements to Perform 14.4 keV $^{57}$Fe Nuclear Resonance?

- Low Emittance Lattice and Top-up Operation
- Undulator Source: High Average Brilliance and High Average Flux
- High Current per Bunch ~ 5-20 mA
- Bunch Width ~ 50-100 ps
- Bunch-to-Bunch Separation ~ 100 – 300 ns ( > 150 ns for $^{57}$Fe)
- Clean Bunches (Purity ~ $10^{-10}$)
- Energy Width of Excitation (14.41 keV) Beam ~ 1-3 meV
- A Detector with a Time Resolution < 1 ns
  → Avalanche Photo Diodes (APDs)
Third-Generation Storage-Ring Facilities in the World
where

One beamline each at four facilities supporting only part of the time due to availability of appropriate bunch pattern.

Over Subscription of Beamlines
Geoscience, High Pressure Research, Biophysics, Material Physics
Comparison of Characteristics of the X-ray Beam from Undulator Sources Used for NRS (around 14 keV) at Third-generation Storage-ring Facilities.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Operating Third-Generation Facilities (APS, ESRF, SPring-8)</th>
<th>PETRA-III *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (GeV)</td>
<td>6.0 - 8.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Maximum Current (mA)</td>
<td>100 - 250</td>
<td>100</td>
</tr>
<tr>
<td>Horizontal Beam Emittance (nm.rad)</td>
<td>3.0-4.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Undulator Length (m)</td>
<td>4.8 - 5.0</td>
<td>20</td>
</tr>
<tr>
<td>Brilliance ph/s/0.1%BW/mrad²</td>
<td>1 – 2 x 10²⁰</td>
<td>2 x 10²¹</td>
</tr>
<tr>
<td>Vertical divergence (mrad)</td>
<td>3 - 5</td>
<td>3</td>
</tr>
<tr>
<td>Average Flux ph/s/meV</td>
<td>4 x 10⁹</td>
<td>2-4 x 10¹⁰</td>
</tr>
<tr>
<td>Bunch length (ps)</td>
<td>70-100</td>
<td>40</td>
</tr>
<tr>
<td>Rep Rate (MHz)</td>
<td>3.8-6.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Photons/meV/bunch</td>
<td>0.6-1.0 x10³</td>
<td>5-8x10³</td>
</tr>
</tbody>
</table>
# Mössbauer Resonant Nuclei

**Radioactive Source:** A

**SR Source:** A

<table>
<thead>
<tr>
<th>H</th>
<th>Li</th>
<th>Be</th>
<th>B</th>
<th>C</th>
<th>N</th>
<th>O</th>
<th>F</th>
<th>Ne</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>Mg</td>
<td>Al</td>
<td>Si</td>
<td>P</td>
<td>S</td>
<td>Cl</td>
<td>Ar</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Ca</td>
<td>Sc</td>
<td>Ti</td>
<td>V</td>
<td>Cr</td>
<td>Mn</td>
<td>Fe</td>
<td>Co</td>
</tr>
<tr>
<td>Rb</td>
<td>Sr</td>
<td>Y</td>
<td>Zr</td>
<td>Nb</td>
<td>Mo</td>
<td>Tc</td>
<td>Ru</td>
<td>Rh</td>
</tr>
<tr>
<td>Cs</td>
<td>Ba</td>
<td>La</td>
<td>Hf</td>
<td>Ta</td>
<td>W</td>
<td>Re</td>
<td>Os</td>
<td>Ir</td>
</tr>
<tr>
<td>Fr</td>
<td>Ra</td>
<td>Ac</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Mössbauer Resonant Nuclei**

- Radioactive Source:
  - A

- SR Source:
  - A
### List of Resonances Using the Third-Generation SR Facilities

<table>
<thead>
<tr>
<th>Z</th>
<th>Isotope</th>
<th>$E_0$ (keV)</th>
<th>$t_0$ (ns)</th>
<th>$\Gamma_0$ (neV)</th>
<th>$\sigma_0$ (kbarn)</th>
<th>$\sigma_0/\sigma_{el}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>$^{181}$Ta</td>
<td>6.214</td>
<td>8730</td>
<td>0.075</td>
<td>1999</td>
<td>12</td>
</tr>
<tr>
<td>69</td>
<td>$^{169}$Tm</td>
<td>8.410</td>
<td>5.89</td>
<td>110</td>
<td>242</td>
<td>7</td>
</tr>
<tr>
<td>36</td>
<td>$^{83}$Kr</td>
<td>9.404</td>
<td>212.1</td>
<td>3.1</td>
<td>1226</td>
<td>152</td>
</tr>
<tr>
<td>26</td>
<td>$^{57}$Fe</td>
<td>14.4125</td>
<td>141.1</td>
<td>4.7</td>
<td>2564</td>
<td>426</td>
</tr>
<tr>
<td>63</td>
<td>$^{151}$Eu</td>
<td>21.514</td>
<td>14.0</td>
<td>47</td>
<td>243</td>
<td>29</td>
</tr>
<tr>
<td>62</td>
<td>$^{149}$Sm</td>
<td>22.496</td>
<td>10.3</td>
<td>64</td>
<td>120</td>
<td>17</td>
</tr>
<tr>
<td>50</td>
<td>$^{119}$Sn</td>
<td>23.871</td>
<td>35.6</td>
<td>26</td>
<td>1381</td>
<td>563</td>
</tr>
<tr>
<td>66</td>
<td>$^{161}$Dy</td>
<td>25.651</td>
<td>42</td>
<td>16</td>
<td>1110</td>
<td>176</td>
</tr>
<tr>
<td>19</td>
<td>$^{40}$K</td>
<td>29.23</td>
<td>6</td>
<td>110</td>
<td>281</td>
<td>1337</td>
</tr>
<tr>
<td>51</td>
<td>$^{121}$Sb</td>
<td>37.133</td>
<td>5</td>
<td>130</td>
<td>195</td>
<td>40</td>
</tr>
<tr>
<td>28</td>
<td>$^{61}$Ni</td>
<td>67.41</td>
<td>7.5</td>
<td>88</td>
<td>721</td>
<td>8100</td>
</tr>
<tr>
<td>93</td>
<td>$^{237}$Np</td>
<td>59.5</td>
<td>98</td>
<td>7</td>
<td>310</td>
<td>115</td>
</tr>
</tbody>
</table>
What future sources are suitable for NRS?

- PETRA III Storage Ring, DESY, Hamburg
- PEP III Storage Ring, SLAC, Stanford
- LCLS SASE-XFEL, SLAC, Stanford
- European SASE-FEL, DESY, Hamburg
- SASE SCSS–SPring8, Japan
- PSI-XFEL, Switzerland
- Seeded XFEL - Future

What are the science directions using NRS at the future facilities?
Essence of a Free-Electron Laser

Incoherent Emission

Each electron is independently radiating and the phases of the electric fields are random

\[ E \propto \sqrt{N_e} \]
\[ \text{Intensity} \propto N_e \]

Coherent Emission

If the electrons are in lock synch and radiate coherently, electric field grows linear with number of electrons

\[ E \propto N_e \]
\[ \text{Intensity} \propto N_e^2 \]
Self-Amplified Spontaneous Emission (SASE)

Spontaneous Emission from High-Density Short Electron Bunch

Phase Separation in Electron Bunches with a Period of the Wavelength of Radiation

Microbunches Behave Like Super Charge and Emit Coherent Radiation with Intensity Enhancement

\[ N_{mb} = \text{number of electrons in coherent volume} \]
Exponential Growth of Intensity in SASE Process

Start up is from noise signal

Microbunching Begins

Saturation

Distance

Log Radiation Intensity

\( e^- \)

\(~100\,\text{fs}\)

\(\lambda\)
Does a SASE Laser Work?

1980  First proposal
1996  First experimental observation
      UCLA: $\lambda = 16 \, \mu m$

Dec. 1999  Argonne: lasing at $\lambda = 523 \, nm$
Sep. 2000  Argonne: saturation at $\lambda = 523 \, nm$
Feb. 2001  VISA/BNL: saturation at $\lambda = 800 \, nm$
Apr. 2001  Argonne: saturation at $\lambda = 265 \, nm$
Sep. 2001  Hamburg -DESY: saturation at $\lambda = 100 \, nm$
Oct. 2001  First user experiments @ DESY- FLASH(TTF)

These early developments led to VUV and X-ray FELs designs and construction.
A SASE FEL amplifies random electron density modulations

The SASE radiation is powerful, but noisy!

One solution: Impose a strong coherent modulation with an external laser source

\[ \tau \ (\text{fs}) \quad \Delta \omega / \omega \ (%) \]
Two of the Many Seeding Schemes
Seeded XFEL (SXFEL)

Two Stage Seeding

FEL Oscillator

Diamond cavity for the X-FEL Oscillator


K.J. Kim and Y. Shvydko

$R_1 \times R_2 \times R_M = 0.91 \quad T_1 \simeq 0.042$
SASE FEL Output in Time and Energy Domain

Seeded FEL Output in Time and Energy Domain

Seeded for fs time domain

Seeded for meV energy domain

Transform Limited 20 meV bandwidth 10-15 keV beam

Courtesy: Bill Graves, David Moncton MIT
### Brightness of Synchrotron Radiation Sources

<table>
<thead>
<tr>
<th>Generation</th>
<th>Source</th>
<th>Electrons</th>
<th>U-Periods</th>
<th>Enhancement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td>Bend Magnet</td>
<td>$\sim N_e$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Wiggler</td>
<td>$\sim N_e$</td>
<td>$\sim N_p$</td>
<td>10</td>
</tr>
<tr>
<td>3rd</td>
<td>Undulator</td>
<td>$\sim N_e$</td>
<td>$\sim N_p^{2-x}$</td>
<td>$10^4$</td>
</tr>
<tr>
<td></td>
<td>(APS, ESRF, SPring-8, ERLs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Next</td>
<td>SASE FEL</td>
<td>$\sim N_{mb}^2$</td>
<td>$\sim N_p^2$</td>
<td>$10^9$</td>
</tr>
<tr>
<td></td>
<td>Seeded FEL</td>
<td>$\sim N_e^2$</td>
<td>$\sim N_p^2$</td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>
Log Average Beam Brightness

Log Peak Brightness

1900 1920 1940 1960 1980 2000 2020 2040

First Generation

Second Generation

Third Generation

SEEDED FELs

SASE FELs


Eric Gerdau et al. Demonstration (1985)

Stan Ruby Proposal (1974)
Gains to be made by SASE FEL and SXFEL over a Third Generation Storage Ring Source

- **FEL:Seeded**
  - $3 \times 10^{16}$ ph/s/mV
  - $10^{10} - 10^{14}$ ph/pulse/mV

- **FEL:SASE**
  - $3 \times 10^{14}$ ph/s/mV
  - $10^{10}-10^{12}$ ph/pulse/mV

- **1-2 x 10^9 ph/s/mV**
- **3.0 x 10^4 ph/pulse/mV**

**Undulator**
LCLS- SLAC - Stanford

14 GeV: 0.1 – 3 nm SASE 2009 - 2010

120 Hz – 135 fs

Euro-XFEL SASE DESY Hamburg

20 GeV: 0.1 – 1 nm SASE XFEL 2012

10 x 3000 Hz - 95 fs

Schenefeld

PSI- XFEL, Switzerland

6 GeV: SASE SCSS–SPring8, Japan

Future X-ray FEL at SPring-8

Target Wavelength 1 Å
6 GeV C-band Accelerator
1 km Site Length
Multiple User Beam Lines

8 GeV SPring-8

Construction of the PSI-XFEL (1 – 0.1 nm Seeded 6 GeV)

10 (100) Hz - 65 fs

o Test of the acceleration concept: 2008 – 2011

o Construction of an X-FEL: 2011-2016
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (keV – 3rd Harmonic)</td>
<td>~ 14.4</td>
</tr>
<tr>
<td>Pulse length (fs)</td>
<td>~ 30-40</td>
</tr>
<tr>
<td>Energy Resolution (meV)</td>
<td>~ 20</td>
</tr>
<tr>
<td>Rep Rate (kHz)</td>
<td>~ 10 – 100</td>
</tr>
<tr>
<td>Photons/meV/pulse</td>
<td>~ $10^{10}$ - $10^{14}$</td>
</tr>
<tr>
<td>Photons/meV/pulse/(1-20 nm)$^2$</td>
<td>~ $10^8$ - $10^{12}$</td>
</tr>
</tbody>
</table>
Revealing the Interactions in Condensed Systems

- Multiphonons & Multimagons ~ 50 – 500 meV
- Pseudogap ~ 30 meV – 300 meV
- Optical phonons ~ 40 – 70 meV
- Magnons ~ 10 meV – 40 meV
- Superconducting gap ~ 1 – 100 meV

- Dynamics of disordered and nano systems
- Boson peak in Amorphous Solids
- Phasons in quasicrystals
- Rotational excitations in liquids
- Soft phonons

- Relaxation dynamics of fluids ~ 1 – 400 neV

- Hyperfine Interactions ~ 1 – 100 neV

peV, feV, aeV, zeV,

Fundamental Physics
Probing Inside a Nanorod with 1-20 nm Beam

SXFEL 100 $\mu$V

1-20 nm spot

25 nm

1 $\mu$m
Measuring the spatially-resolved diffusion dynamics in confined systems (nanofluidics)

Röhlsberger et al NIM A394, 251 (1997)

Coherent forward scattering

Before atomic diffusion
Scattered radiation in phase

Atomic jump during nuclear life-time leads to dephasing scattered radiation

Molecular diffusion through Carbon Nanotube
Partial Phonon Density Of States of Iron Films

Fe films deposited on W(110)
Transition from the bulk to a single iron monolayer

S. Stankov, R. Röhlsberger, T. Slezak, M. Sladecek, B. Sepiol, G. Vogl, A. I. Chumakov, R. Rüffer, N. Spiridis, J. Lazewski, K. Parlinski, and J. Korecki,

ESRF Highlights 2006

Can One Probe Far-From-Equilibrium Dynamics?  
*Processes in the ps to µs time domain*

**Pump:** Electric, Magnetic, Laser, THz, Shock Wave, etc  
**Probe:** NRS (both coherent & incoherent elastic and incoherent inelastic)

Can a single shot characterize the far-from-equilibrium processes?
**Future of Ultimate Metrology**

\[ ^{45}\text{Sc} \]

\[ 3/2^+ \rightarrow 12.5 \text{ keV} \rightarrow 7/2^- \]

\[ T_{1/2} = 318 \text{ ms} \]

\[ \alpha \sim 400 \]

- \[ ^{45}\text{Sc}(2.8 \text{ MeV p})^{45m}\text{Sc} \]

- \[ ^{229}\text{Th} (7340 \text{ yrs}) \]

\[ 9/2^+ \rightarrow 54.70 \rightarrow 25.31 \rightarrow 97.13 \]

\[ 7/2^+ \rightarrow 29.39 \rightarrow 71.82 \]

\[ 7/2^+ \rightarrow 42.43 \rightarrow 42.63 \rightarrow 29.19 \]

\[ 5/2^+ \rightarrow 0.0076 \rightarrow 29.18 \]

- \[ ^{3/2^+} \rightarrow 0.0076 \text{ keV} \rightarrow 5/2^+ \rightarrow 0 \]

\[ T_{1/2} = 26,000 \text{ s} \]

\[ \alpha \sim 1000 \]

K W Jones et al. PR 148, 1148 (1966)

Beck et al. PRL 2007
<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$^{57}$Fe</th>
<th>$^{45}$Sc</th>
<th>$^{229}$Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abundance, $\alpha$ (%)</td>
<td>Stable (2.14%)</td>
<td>Stable (100%)</td>
<td>7340 yrs (100%)</td>
</tr>
<tr>
<td>$E_o$</td>
<td>14.4 keV</td>
<td>12.4 keV</td>
<td>7.6 eV</td>
</tr>
<tr>
<td>$I_g - I_e$</td>
<td>1/2 - 3/2 -</td>
<td>7/2+ - 3/2 -</td>
<td>5/2+ - 3/2+</td>
</tr>
<tr>
<td>Multipolarity</td>
<td>M1</td>
<td>M2</td>
<td>M1</td>
</tr>
<tr>
<td>$T_{1/2}$</td>
<td>99 ns</td>
<td>318 ms</td>
<td>26,000 s</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>4.6 neV (0.96 mm/s)</td>
<td>1.46 feV (35 nm/s)</td>
<td>17.5 zeV (0.7 pm/s)</td>
</tr>
<tr>
<td>$Q = \Gamma/E_o$</td>
<td>3.2 x 10^{-13}</td>
<td>1.18 X 10^{-16}</td>
<td>2.30 X 10^{-21}</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>8.21</td>
<td>~ 413</td>
<td>~1000</td>
</tr>
<tr>
<td>$\sigma_0 \text{ cm}^2$</td>
<td>2.56 X 10^{-18}</td>
<td>1.98 X 10^{-20}</td>
<td>2.11 X 10^{-14}</td>
</tr>
<tr>
<td>$\sigma_0/\sigma_{el}$</td>
<td>426</td>
<td>4</td>
<td>&lt; 20</td>
</tr>
<tr>
<td>Unit Resonant Absorption Thickness (f=1, $\alpha = 100%$)</td>
<td>4.6 (µm) (3.8 X 10^{19}/cm^2)</td>
<td>12.0 (µm) (4.8 X 10^{19}/cm^2)</td>
<td>1.6 (nm) (4.9 X 10^{15}/cm^2)</td>
</tr>
</tbody>
</table>
Coherent Nuclear Excitation of $^{229}$Th

**Issues:**
- Hyperfine interaction of ground and excited moments with surrounding electronic and nuclear moments
- Elimination of all sources of line broadening
- Detection of 7.6 eV radiation

**Approaches:**
- Intense laser excitation and detection through laser spectroscopy of atomic transitions (Habs: this symposium)
- Excitation of 29.19 keV state with x-ray SR/ XFEL to populate 7.6 eV state (Argonne)
- Preparation of stripped nuclei in storage-cooler rings or in ion-traps (Suggestion: P. Kienle, TU Munich)
Optical Detection Scheme: Test of Fundamental Physics

7.6 eV Resonance

\[ |I_g, J_2\rangle \rightarrow |I_{ex}, J_2\rangle \quad \omega_1^{\text{opt}} \]

\[ |I_g, J_1\rangle \rightarrow |I_{ex}, J_1\rangle \quad \omega_2^{\text{opt}} \]

\[
H_{I_g, I_e, J_{1,2}, L_{1,2}, S_{1,2}}^{hf} = \bar{A} \cdot (\bar{I}_{g,e} \cdot \bar{J}_{1,2})
\]


Dietrich Habs, The Lowest Nuclear Transition in $^{229}$Th: This Symposium Presentation
Gravitational Red Shift Scheme

\[ \Delta E_g = E_0 g H/c^2 \text{ (eV)} \]

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$^{57}$Fe</th>
<th>$^{45}$Sc</th>
<th>$^{229}$Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E_g$ (eV)</td>
<td>(3.54 \times 10^{-11})</td>
<td>(1.35 \times 10^{-15})</td>
<td>(8.3 \times 10^{-21})</td>
</tr>
<tr>
<td>H</td>
<td>22.6 m</td>
<td>1 mm</td>
<td>10 μm</td>
</tr>
<tr>
<td>$\Delta E_g/\Gamma$ (%)</td>
<td>0.8</td>
<td>92</td>
<td>47</td>
</tr>
</tbody>
</table>

Jefferson Tower
Harvard

Th$^{4+}$ (6d$^0$)
Summary:

1. Metrology, Fundamental Physics, Cosmology, Gravitation

<table>
<thead>
<tr>
<th>Nobel Prize</th>
<th>Laureate(s)</th>
<th>Technique</th>
<th>Resolution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>Mössbauer</td>
<td>NRS</td>
<td>$1: 10^{13}$</td>
<td>Numerous</td>
</tr>
<tr>
<td>2005</td>
<td>Hall and Hänch</td>
<td>Frequency Comb</td>
<td>$1: 10^{15}$</td>
<td>Atomic Clock Metrology</td>
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<tr>
<td>????</td>
<td>??????</td>
<td>NRS Hyperfine Comb</td>
<td>$1: 10^{16}$</td>
<td>Fundamental Science Cosmology Gravitation</td>
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<tr>
<td>????</td>
<td>??????</td>
<td>NRS</td>
<td>$1: 10^{21}$</td>
<td></td>
</tr>
</tbody>
</table>

Flambaum, PRL 97, 092502(2006); Burvenich et al, PRL 96, 142501 (2006)

2. Nonlinear Quantum Optics
## Non-linear Nuclear Quantum Optics

<table>
<thead>
<tr>
<th>Incident Process</th>
<th>Generated Frequencies</th>
<th>Non-linear Frequencies</th>
<th>Susceptibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parametric conversion</td>
<td>$\omega_\pi$</td>
<td>$\omega_\sigma$, $\omega_1$ ($\omega_\pi = \omega_\sigma + \omega_1$)</td>
<td>$\chi^{(2)}(\omega_2; -\omega_3, \omega_1)$</td>
</tr>
<tr>
<td><strong>2nd (and Higher) Harmonic Generation</strong></td>
<td>$\omega_1$</td>
<td>$\omega_2$ ($\omega_2 = 2 \omega_1$)</td>
<td>$\chi^{(2)}(\omega_2; \omega_1, \omega_1)$</td>
</tr>
<tr>
<td>Mixing</td>
<td>$\omega_1$, $\omega_2$</td>
<td>$\omega_3$ ($\omega_3 = \omega_1 + \omega_2$)</td>
<td>$\chi^{(2)}(\omega_3; \omega_2, \omega_1)$</td>
</tr>
<tr>
<td>Intensity dependent index of refraction</td>
<td>$\omega_1$</td>
<td>$\omega_1$</td>
<td>$\chi^{(3)}(-\omega_1; \omega_1, -\omega_1, \omega_1)$</td>
</tr>
<tr>
<td>2 photon absorption</td>
<td>$\omega_1$</td>
<td>-</td>
<td>$\chi^{(3)}(\omega_1; \omega_1, -\omega_1, \omega_1)$</td>
</tr>
</tbody>
</table>

\[
[c_{k_1} + c_{k_2}] = \omega_M = 3 \times 7.2 \text{ keV} = 14.4 \text{ keV}
\]

**XFEL field strength is $\sim 1 \text{ V.A}^{-1}$ $\rightarrow 10 \text{ keV}**

*(Seb Doniach JSR 7, 216,2000)*
Extra Reading

TDR XFEL workshop series
Nuclear Resonant Scattering at the TESLA XFEL

Edited by H. Franz and U. van Burck
February 25, 2001

Scientific opportunities in nuclear resonance spectroscopy from source-driven revolution
G. K. Shenoy · R. Röhlsberger
(in press)
Thank You